10.5 - Day 2 - Partial Fraction Decomposition

Case 3: If Q contains a nonrepeated irreducible quadratic factor of the form ax^2+bx+c, then, in the partial fraction decomposition of $\frac{P}{Q}$, allow for the term:

$$\frac{Ax+B}{ax^2+bx+c}$$ where A + B are to be determined.

4.) Write the partial fraction decomposition of $\frac{3x-5}{x^3-1}$

① Factor denom: $x^3-1=\boxed{(x-1)(x^2+x+1)}$

$a^3-b^3=(a-b)(a^2+ab+b^2)$ see that it has a nonrepeated linear factor of $(x-1)$ and a nonrepeated irreducible quadratic factor of x^2+x+1.

↳ allow for $\frac{A}{x-1}$ by case 1, and $\frac{Bx+C}{x^2+x+1}$ by case 3

② $$\frac{3x-5}{x^3-1}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$$

③ Clear fractions (get rid of denominators) by mult. each side by $(x-1)(x^2+x+1)$

$$(x-1)(x^2+x+1)\left[\frac{3x-5}{x^3-1}\right]=\left[\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}\right](x-1)(x^2+x+1)$$

↳ $3x-5=A(x^2+x+1)+(Bx+C)(x-1)$

$3x-5=Ax^2+Ax+A+Bx^2-Bx+Cx-C$

$3x-5=x^2(A+B)+x(A-B+C)+(A-C)$

<over>

#4 Continued

$0 = A + B$ coef. of x^2

$3 = A - B + C$ coef of x

$-5 = A - C$ coef of x^0 → $C = A + 5$ (plug into 2nd equ)

$3 = A + B + (A+5)$

$-2 = 2A - B$

$+ \; 0 = A + B$

$\frac{-2}{3} = \frac{3A}{3}$ → $A = -\frac{2}{3}$

plug A into $0 = A + B$

$0 = -\frac{2}{3} + B$

$\frac{2}{3} = B$

$3 = \left(-\frac{2}{3}\right) - \left(\frac{2}{3}\right) + C$

$3 + \frac{4}{3} = C$ → $\frac{13}{3} = C$

(4) $\frac{3x-5}{x^3-1} = \frac{-\frac{2}{3}}{x-1} \quad \frac{\frac{2}{3}x + \frac{13}{3}}{x^2+x+1}$

Case 4: If Q has repeated irreducible quadratic factor, then:

$\frac{A_1 x + B_1}{ax^2+bx+c} + \frac{A_2 x + B_2}{(ax^2+bx+c)^2} + \ldots + \frac{A_n x + B_n}{(ax^2+bx+c)^n}$

5.) Write the partial fraction decomposition of $\frac{x^3+x^2}{(x^2+4)^2}$

$\frac{x^3+x^2}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2}$

- Clear fractions & obtain $x^3+x^2 = (Ax+B)(x^2+4) + Cx + D$

$x^3 + x^2 = Ax^3 + 4Ax + Bx^2 + 4B + Cx + D$

$x^3 + x^2 = Ax^3 + Bx^2 + (4A+C)x + 4B + D$

equating coefficients we get $1 = A$, $1 = B$, $0 = 4A + C$, $0 = 4B + D$

↳ $0 = 4A + C \rightarrow 0 = 4(1) + C \rightarrow -4 = C$

$0 = 4B + D \rightarrow 0 = 4(1) + D \rightarrow -4 = D$

• $\frac{x^3+x^2}{(x^2+4)^2} = \frac{x+1}{x^2+4} + \frac{-4x-4}{(x^2+4)^2}$